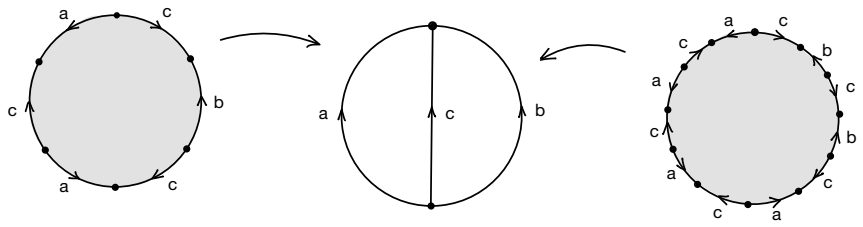
RICE UNIVERSITY TOPOLOGY QUALIFYING EXAM - MAY 2023

This is a 4 hour, closed book, closed notes exam. **Justify all of your work** as much as time allows. Write and sign the Rice honor pledge at the end of the exam.

Honor pledge: On my honor, I have neither given nor received any unauthorized aid on this exam.

Note: All homology/cohomology groups are taken with $\mathbb{Z}$ coefficients.

1. Let W be the space obtained from the graph with edges a, b, c by attaching two disks as indicated in the figure below. Find a presentation for $\pi_1 W$ and prove that it is infinite.



2. Let X and Y be the wedge products, $X = \mathbb{RP}^3 \vee \mathbb{RP}^3$ and $Y = S^1 \vee S^1$. Prove that any map $f\colon X \to Y$ is null-homotopic.

3. Up to isomorphism, how many connected, regular, five sheeted covers of the three torus $\mathbb{T}^3$ are there?

4. Let Z be the wedge product of a circle and a compact, connected, orientable surface (without boundary) of genus 2. Prove that Z is not homotopy equivalent to a compact surface (without boundary).

5. Suppose $K \subset S^3$ is an embedded circle. Compute $H_p(S^3 - K)$ for all $p \geq 0$. You may assume that K has a neighborhood $U \subset S^3$ such that $(U, K) \cong (S^1 \times \mathbb{D}^2, S^1 \times \{0\})$.

6. (a) What is the cohomology ring of $\mathbb{CP}^n$?
   (b) Prove that there is no degree -1 map from $\mathbb{CP}^n$ to itself when $n > 0$ is even.
   (c) Construct a degree -1 map from $\mathbb{CP}^n$ to itself when $n > 0$ is odd.