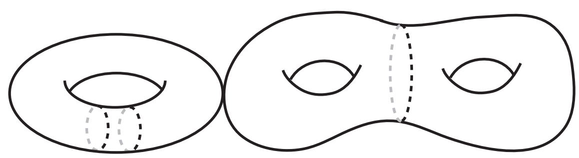
RICE UNIVERSITY TOPOLOGY QUALIFYING EXAM - JANUARY 2014

This is a FOUR HOUR closed book, closed notes exam. JUSTIFY all of your work, as much as time allows. At the end SIGN THE RICE HONOR PLEDGE and turn in both your exam paper and this paper.

1. Let $W = S^1 \vee S^1$ be the wedge of 2 circles. Describe three (distinct up to covering space equivalence) connected 3-fold covering spaces of W, including one irregular cover. Don't forget to describe the covering map. In each case, give the group of covering transformations, say whether or not the covering is regular and give the corresponding subgroup of $\pi_1(W)$.

2. Suppose Y is the space obtained from the wedge of a torus and a genus two surface (pictured below) by adjoining three 2-dimensional disks. Two of the disks are adjoined along meridional circles of the torus, and the third is adjoined along the "waist" circle of the genus two surface. The three attaching circles of these disks are the dashed circles. The three disks are not pictured.



- (a) Compute $\pi_1(Y)$.
- (b) Compute $H_p(Y; \mathbb{Z})$ for all p.
- (c) Show that Y has the structure of a CW complex.
- (d) Calculate a cellular cochain complex with $\mathbb{R}$-coefficients for Y and use it to calculate $H^*(Y; \mathbb{R})$.

3. Give an example for each of the following or explain "such an example does not exist because...". In any case, explain your answers fully.

- (a) two spaces with isomorphic integral homology groups but non-isomorphic π_1.
- (b) two spaces with isomorphic integral homology groups but non-isomorphic cohomology groups.
- (c) two spaces with isomorphic π_1 and isomorphic integral homology groups which are NOT homotopy equivalent;
- (d) two spaces that are homotopy equivalent but are not homeomorphic

4. Prove that, for any continuous map $f : \mathbb{C}P(4) \to S^3 \times S^5$, the induced map f_* on $H_8(-; \mathbb{Z}_5)$ is the zero map.

5. Prove that if M is a compact, contractible, orientable n-manifold ($n \geq 1$), then ∂M is a homology $(n-1)$-sphere, that is, $H_i(\partial M; \mathbb{Z}) \cong H_i(S^{n-1}; \mathbb{Z})$ for all i.