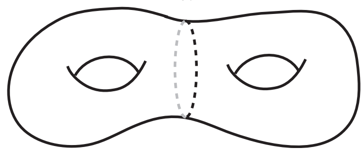
RICE UNIVERSITY TOPOLOGY QUALIFYING EXAM - JANUARY 2012

This is a 3-hour closed book, closed notes exam. Please show all of your work.

1. a) Discuss all connected covering spaces of $S^1 \times \mathbb{RP}(3)$. Specifically discuss "how many" there are, regular/irregular, how you know there aren't any more, their groups of covering translations, possibly include pictures.

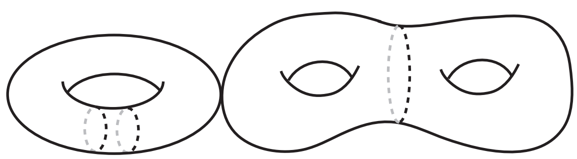
   b) Calculate $\pi_2(S^1 \times \mathbb{RP}(3))$.

2 . Let A be the surface of genus 2 show below and let α be the dashed curved ("the waist curve").



Let B be another surface of genus 2 with waist curve β. Let X be $A \cup_f B$ identifying the two curves α and β by a homeomorphism $f : \beta \to \alpha$.
(a) Give a presentation for $\pi_1(X)$.
(b) Calculate $H_p(X)$ for all p.

3 . Suppose Y is the space obtained from the wedge of a torus and a genus two surface (pictured below),by adjoining three 2-dimensional disks, two along meridional circles of the torus, and the third along the "waist" circle of the genus two surface. The three attaching circles of these disks are the dashed circles. The three disks are not pictured.



a. Show that Y has the structure of a CW complex and derive the corresponding cellular chain complex.
b. Compute $H^p(Y; \mathbb{Z})$ for all p.

4 . Prove that any continuous map $f : \mathbb{C}\mathrm{P}^4 \to S^3 \times S^5$ induces the zero map on $H_8(-; \mathbb{Z})$.

5. Describe a 1-dimensional CW-complex on which the symmetric group S_3 acts freely and properly discontinuously. Also describe the quotient space.

6. Let M be a compact, orientable 3-dimensional manifold with boundary. Suppose that $H_1(M; \mathbb{Z})$ is a finite group of odd order. Prove that the boundary of M is a disjoint union of 2-spheres. (Hint: Find $H_1(\partial M)$).