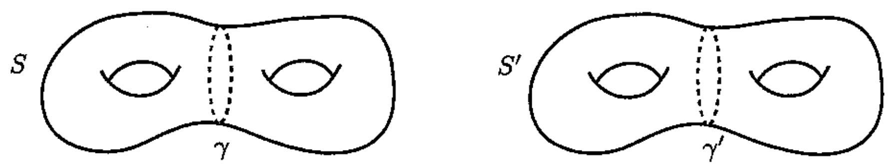
RICE UNIVERSITY TOPOLOGY QUALIFYING EXAM - MAY 2008

This is a 3-hour closed book, closed notes exam. Please show all of your work.

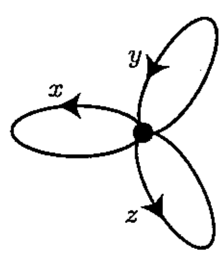
1. Let $W = S^1 \vee S^1$ be the wedge of 2 circles. Describe four distinct connected 3-fold covering spaces of W including at least one irregular cover. In each case, give the group of covering transformations, say whether or not the covering is normal (regular) and give the corresponding subgroup of $\pi_1(W)$.

2. Let S and S' be orientable surfaces of genus 2 as show below. Let X be the space obtained from $S \sqcup S'$ by identifying the circle γ in S to the circle γ' in S'.



a) Give a presentation for $\pi_1(X)$.
b) Compute $H_p(X)$ for all p.

3. Let $W = S^1 \vee S^1 \vee S^1$ as shown below. Let x, y, z be the three loops indicated going around the first, second, third circles respectively. Let $X = W \cup_{f_1} e_1^2 \cup_{f_2} e_2^2$ be the space obtained from W by adjoining one 2-cell via the map f_1 which forms the loop $xyx^{-1}zy^{-1}z^{-1}$; and another 2-cell via the map f_2 which forms the loop x^7.



a) Give a cellular chain complex for X (including the boundary maps).
b) Compute $H_p(X; \mathbb{Z}_2)$ for all p.
c) Compute $H^p(X; \mathbb{Q})$ for all p without using a Universal Coefficient Theorem.

4. Suppose $f : I \to Y$ is a path in Y and $g : I \to I$ is a continuous map such that $g(0) = 0$ and $g(1) = 1$. Prove that f is homotopic to $f \circ g$.

5. Let X and Y be compact, connected, oriented n-dimensional manifolds without boundary and let $f : X \to Y$ be a continuous map. Suppose $\beta_p(X) < \beta_p(Y)$ for some $p > 0$.

a) Prove that $f^* : H^p(Y; \mathbb{Q}) \to H^p(X; \mathbb{Q})$ has a non-trivial kernel.
b) Show that f is a degree zero map.