

Syllabus for the Qualifying Exam in Algebra

Rice University Department of Mathematics

Spring 2008

I) Group theory

1. Definitions and basic properties

- Subgroups
- Homomorphisms
- Direct and semidirect products
- Normal subgroups and quotients

2. Examples of groups

- Symmetry groups
- Permutation groups
- Cyclic and dihedral groups
- Finite groups represented as matrices
- Linear groups (SL_2 , SO_2 , etc.)
- Finitely-generated abelian groups
- Free groups
- Presentations of groups via generators and relations

3. Groups acting on sets

- Conjugacy classes
- Sylow Theorem
- Solvable groups

II)Elementary ring theory

1. Definitions and basic properties

- Ideals
- Homomorphisms
- Quotient rings
- Polynomials
- Maximal and prime ideals

2. Factorization

- Irreducible elements
- UFD's and PID's
- Factorization over rings of integers

III)Advanced linear algebra

1. Quadratic and bilinear forms

2. Jordan and rational canonical form

3. Finitely-generated modules over PID's

4. Hom and dual spaces

5. Tensor algebras, exterior algebras, symmetric algebras

IV)Field theory

1. Field extensions, algebraic and transcendental elements

2. Finite fields

3. Splitting fields

4. Constructible numbers

5. Galois groups

6. Cyclotomic extensions

7. Structure of quadratic, cubic, and quartic extensions

8. Solvability in radicals

V)Commutative algebra

1. Gröbner bases with application to elimination and ideal membership

2. Hom and tensor products over general rings;

3. Tensor, exterior and symmetric algebras over rings

4. Noetherian and Artinian rings

5. Integral extensions, rings of integers over number fields

6. Nullstellensatz

7. Local rings and localization of modules

VI)Homological algebra and structure of modules

1. Complexes and exact sequences

2. Free, projective, flat, and injective modules

3. Ext and Tor